



Overview of optimizing on-line display-ad allocation on the basis of advertiser budget constraints

Mr. Parag B. Patil¹ Mr. Ashish S. Awate² Mr. Abhijeet S. Joshi³

¹M.E., Dept. of Comp. Sci. & Engg
Pankaj Laddhad Institute of Technology, Buldhana, India

²Asst.Prof., Dept. of Comp. Sci. & Engg
STC, School of Engineering and Research Technology, Shegaon, Buldhana (M.S.)

³M.E., Dept. of Comp. Sci. & Engg Anuradha Engineering College, Chikhali, India

ABSTRACT

It is common among on-line publishers to attract their visitors by displaying advertisements. To do so, they have the option to use systems called display-ad exchanges to help decide which advertisements are shown to each visitor. The key challenge is to allocate advertisements to viewers in a real time setting. This paper adopts a model that optimizes how the displayed exchange spends the budget of advertisers in order to maximize the revenue of the publisher. This problem is virtually unaddressed in literature. The model is constructed by combining an off-line linear programming model with a linear regression model for web traffic prediction. This combination renders a solution from which it is possible to measure return-on-investment values that can be used by the display-ad exchange to increase the publisher revenue. The paper adopts greedy Baseline algorithm that simulates key characteristics of a real display-ad exchange. Comparing the return-on investment heuristic with the Baseline for a set of real data, shows a increase in publisher revenue. This increase is achieved by spending advertiser budgets more efficiently. The off-line linear programming model shows theoretical revenue improvements in the region, and that this depends on how many advertisers completely consume their budgets.

Keyword -- Display-ad exchanges, a greedy Baseline algorithm, return-on-investment, adword, off-line linear programming

I. INTRODUCTION

On-line publishers can monetize their visitors by displaying advertisements. They have the option to use advanced systems called ad exchanges to help them decide which advertisements are shown to their visitors.

There exist two main types of ad-exchanges:

- Search- ad exchanges like Google AdWords[8], pick which advertisements are shown to each visitor based on keywords provided by the visitor. Most literature available focuses on search-ad exchanges.
- Display-ad exchanges decide what advertisements are shown to each visitor based on relevant available parameters such as: characteristics of the advertisements and the advertisers; contextual information of the website; and visitor behavior and demographics if available.

This paper is about demonstrating the concept of maximizing the revenue of publishers that use displayed exchanges. In a display-ad exchange the publisher has a set of placements on his website; advertisers bid money to have their advertisements shown on the publisher's placements; the ad exchange picks the winning bidder and the winner gets their advertisement shown. The bid is what renders revenue to the publisher. For both types of ad exchanges there exists literature attempting to find optimal bidding strategies for the advertisers. In this paper one can diverge from the perspective of the advertiser and assume the perspective of the publisher, something not commonly done.

A. Problem definition

The publisher wants to maximize his revenue. The revenue comes from the advertisers who place bids to have their advertisements shown. The advertiser pays the publisher either per thousand views of an advertisement, when a visitor clicks the advertisement, or performs some other action on the advertiser's site after viewing or clicking on the advertisement. The advertisers often also have a limited budget to spend and as the ad exchange greedily picks the advertisement that has the highest expected revenue, this does not take into account the fact that it might be more profitable to save the advertiser's budget for a different visitor, placement or time when the expected revenue of this advertisement might be higher. Paper look at the problem of advertiser budget optimization from the perspective of the publisher. Optimizing how the publisher utilizes the budgets of advertiser's means that it want to consume the budget of the advertisers using as few views as possible. The idea



is that some advertisers will completely consume their budgets during some time period, and some advertisers will not. By reducing the amount of web traffic used by advertisers consuming their budgets, the remaining advertisers will have more web traffic available to them. The effect is increased revenue for the publisher.

B. Difficulties

The problem is difficult for a set of reasons:

- (i) publishers receive large amounts of web traffic each day (tens or hundreds of millions of advertisements must be displayed), and each visitor must be served in real-time;
- (ii) publishers have a large set of placements and advertisers, making the problem of picking advertisements for placements combinatorial difficult;
- (iii) Visitors arrive to the publisher in an on-line fashion. The pattern in which the visitors arrive is difficult to predict. Consequently, this makes it difficult to decide if to show an advertisement now, or save it for later;
- (iv) ad exchanges are dynamic systems: advertisers, placements and bids can enter and leave the system at any time.

The problem is both relevant and important since even a slight increase in publisher revenue subsequently renders an increase in revenue for the ad exchange provider. It also strengthens the competitive edge for the ad exchange, making the system more marketable and appealing to on-line publishers.

C. A Budget Optimization Model

This paper will give a complete budget optimization model, something that has not been done before. It will show a theoretical increase in revenue and an on-line heuristic that increases the revenue in a real-time production-like system using real data. Furthermore, it show that the advertisers that completely consume their budget do so in fewer advertisement views, indicating that it is also better at choosing the right target audience to show the advertisements to. The budget optimization model is plug-in by nature, meaning that it does not depend on the inner workings of the ad exchange to be usable. Since the output of the LP is a static allocation of web traffic, even if it is an optimal solution for the prediction, it is not something that is immediately usable in the on-line scenario. Since placements, orders and bids can be added, removed and changed at any time during the days, the heuristic must be flexible enough to handle these changes.

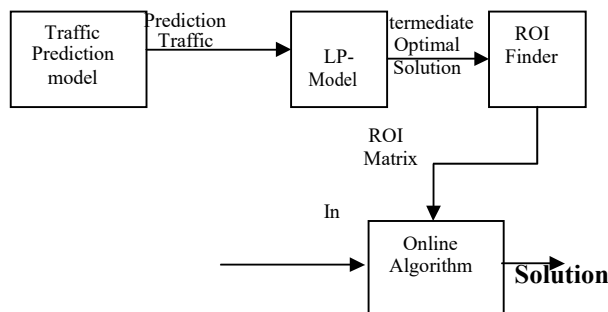


Fig 1: The budget optimization model.

By taking the static LP solution for the predicted web traffic, and measuring return on investment (ROI) values, one can use the ROI values to adjust the bids of advertisers for use in the on-line scenario. This approach is flexible enough since it does not depend on some static allocation, making it useful in practice. The complete model can be seen in Figure 1. The budget optimization model reduces the amount of web traffic used by advertisers completely consuming their budgets, and increases the total revenue of the publisher.

II. BACKGROUND

A. Domain description

At the highest level in the ad exchange, it has publishers. Publishers own websites, and they define areas on these websites where they allow the ad exchange to place advertisements. These areas are called placements. It is the job of the ad exchange to decide to which visitor and on what placement an advertisement is shown. Advertisements that can be shown on placements are called materials.

i) Materials

Materials are advertisements that can be allocated to placements by the ad exchange. There are two different types of materials:

Creative materials are images or something more dynamic, such as an Adobe Flash animation. Creative



materials cover whole placements, i.e., it can fit at most one Creative material per placement.

Text materials are a short piece of text that may be coupled with a small static image. Depending on the size of placements, it is possible to fit many text materials on one placement. Each placement has its own text material capacity defining how many text materials can fit at the same time. As not all placements accept text materials, the text material capacity can be zero.

ii) *Impressions*

When a visitor downloads (views) the publishers website, this is called an impression. In the context of this paper one will count placement impressions. That is, if a website has two placements, and a single visitor enters the site, anybody will count two impressions, one for each of the placements. Placement impressions are independent of how many materials that are actually appearing on the placement. This allows us to abstract away from the concept of websites, and only consider a set of placements. This is useful since publishers can have multiple websites.

iii) *Orders*

Advertisers supply the publisher with materials by placing orders. An order is a set of materials coupled with bids (how much the advertiser is willing to pay to have their materials shown) and a budget. The ad exchange guarantees that the advertiser will never be charged more money than his budget or his bid.

iv) *Expected Cost per Milli*

When choosing which material(s) to display, the ad exchange operates in terms of estimated Cost per Milli (eCPM)[1]. The eCPM is the expected revenue from showing a material one thousand times. The eCPM for each material is calculated for each impression by combining the bids of the advertisers with the likelihood that a click (or some other action that the advertiser is paying for) is performed following display of the material. For the remainder of this paper, one can use eCPM as a direct representation of publisher revenue. The ad exchange picks which materials to put on each placement in a real-time environment. This means that for each impression the system picks the highest rated eCPM material in sequence until all placements have materials assigned to them. Because orders have limited budgets, this is not an optimal heuristic for picking materials.

B. Problem definition

More precisely: to maximize publisher revenue within the context of an existing ad exchange. Paper want to find a heuristic that works for the case where impressions arrive to the publisher in an on-line fashion, and must decide per impression what material is shown on what placement in order to maximize the total revenue of the publisher subject to taking careful consideration of how advertiser budgets are spent. It has identified a set of sub-problems, or parts, that each needs to be solved.

1. A formal Baseline for measuring optimization performance.
2. Off-line budget optimization. It is important to note that the output of this part can not only be a revenue figure, but must be able to provide an optimal placement $\times$ material $\times$ time allocation.
3. Prediction of placement impressions (traffic prediction).
4. Development of a heuristic for the on-line budget optimization.

To solve the problem, one can build a complete budget optimization model. The parts of the model should be as interchangeable as possible. Further, the runtime of the budget optimization model should preferably be in the region of minutes Figure 2 shows how adopted model will fit into the ad exchange.

C. Limitations

The scope of this is quite broad. While the main objective is to maximize publisher revenue subject to advertiser budget constraints, it does have exchange system. That is, it is not attempting to re-invent an ad exchange, and it is not interested in how the eCPM predictions are done. Hence assume that the eCPM predictions are perfect and always available. The choice of methods used for implementing each of the parts of the budget optimization model is subject to two main motivations:

- (1) They are fairly quick and intuitive to develop and implement; and
- (2) If an initial approach gives promising results, it provides a motivation for adding more sophistication later.

As stated, the runtime of the parts are of significance, but it is not something that one has to spend a great amount of effort trying to reduce. As long as the runtime has been reasonable, it has been content.

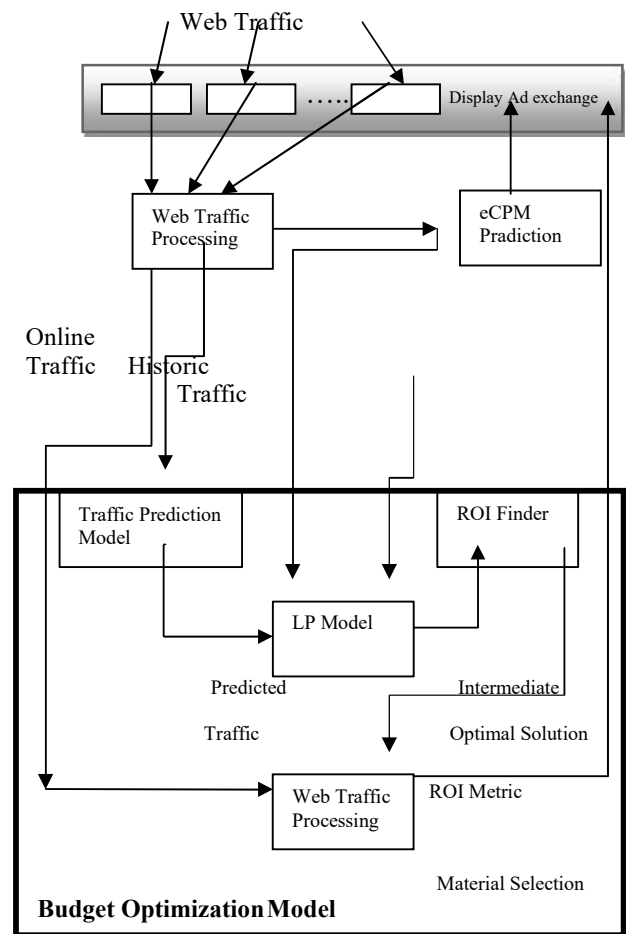


Fig 2: The budget optimization model within the context of an ad exchange. In this paper develop the parts in the outlined box labeled “Budget optimization model”[3][6].

III. DATA MODEL FORMAT

Paper proposed a unified data model, as seen in Figure 3, that can be used as input by the methods presented later in this . The data model captures essential characteristics of a real ad exchange. In the data model, the publisher is the user the ad exchange. The publisher has a set of placements and a set of orders provided by advertisers.

A. Orders

Each order contains a set of materials, bids and a budget. The materials are advertisements that the advertiser wants to show visitors and can be a combination of both creative and text materials. Depending on the nature of the advertisement campaign the advertiser wants to run, the size of the budgets can vary quite a lot. Further, some advertisers choose to define daily budgets (i.e., a daily spending limit) in order to control how the total of their budgets are being spent, e.g., to gain control of the length of an advertisement campaign. If an order has both a total and a daily budget, the data-model will consider the minimum of the two as the actual budget. In this data-model it do not have direct access to the bids, instead one can make decisions based on the eCPM predictions, which incorporate the bids.

eCPM predictions

The ad exchange combines materials, bids, placements, time of day and other parameters to produce an eCPM lookup-table. At any time it can ask the ad exchange what the eCPM for a given material and placement combination is. The eCPMs are treated as a direct representation of publisher revenue for showing a material on a placement, and are used to decide what materials are shown for each impression.

Observed impressions

It has a set of observed impressions for each placement and time. In the online scenario this is undefined, but



from recorded traffic one can read how many impressions each placement received during previous hours. These recorded impressions are ordered, so it is possible to trace exactly when each placement received impressions. This is useful in the Baseline algorithm, as it allows the algorithm to perform a trace of observed placement impressions, but substituting the real ad exchange with method of material selection.



IV. FORMAL BASELINE ALGORITHM

Previously it is defined the input data model. Here it will present a greedy Baseline algorithm [7] that simulates a real ad exchange. See that the Baseline algorithm is unable to consider high eCPM impressions arriving late in the day due to over-spending of order budgets early in the day. This motivates the development of a budget optimization model. The revenue of the Baseline algorithm will be used as a revenue lower bound.

A. An ad exchange algorithm

It want Baseline algorithm[1] to parse impressions in a greedy on-line fashion, and for each placement impression pick the highest eCPM material m , if the order containing m has enough budget left. Paper will adopt a simple, greedy algorithm that illustrates the behavior of the ad exchange. It is defined in Algorithm 1.

Algorithm 1 Ad exchange algorithm

I := stream of placement impressions
 $eCPM_{m,i}$:= revenue of showing material m for impression i
 M := Set of materials.
 $order(m)$:= remaining budget for order containing material m
for all $i \in I$ **do**
 $sort(M)$ w.r.t. $eCPM_m \square M, i$
for all $m \in M$ **do**

Algorithm 1 sorts the materials by eCPM highest to lowest for each placement, and picks the first material with enough budgets left. While this end for end for serves to give an easy to grasp representation of the problem, it does not consider text materials, and it is not adapted to data model format.



B. The Baseline algorithm

Algorithm 1 picks the highest eCPM material m as long as the order of m has budget left. It parses all impressions in sequence. However, it needs more domain constraints in Baseline to be able to parse the data have available. Most importantly, one has to be able to consider creative materials as well as text materials. Adding these considerations will further enhance the relevance of the results of the Baseline as well. Recall that eCPM predictions and placement impressions are aggregated by hour. The ordering of the impressions is preserved, but they are divided into a separate set for each time-period in the system. Algorithm 1 to consider these additions in Algorithm 2

Algorithm 2 The Baseline algorithm. $T :=$ ordered set of time-periods.

$I_t :=$ stream of ordered placement impressions divided into hours $t \in T$.

placement(i) := placement for impression i . $P :=$ set of placements.

eCPM $_{t,p,m}$:= revenue of showing material m on placement p at time t .

order(m) := remaining budget for order containing material m .

$C :=$ set of creative materials. $T :=$ set of text materials.

$C_p :=$ text material capacity for placement p . for all $t \in T$ do

for all $p \in P$ do

C_p sort C w.r.t. eCPM $_{t,p,m}$ T_p sort T w.r.t. eCPM $_{t,p,m}$ end for

for all $i \in I_t$ do

$p \leftarrow \text{placement}(i)$

$c \leftarrow \text{takeWithBudgetLeft}(C_p, 1)$ $ts \leftarrow \text{takeWithBudgetLeft}(T_p, C_p)$

if eCPM $_{t,p,c} \geq \max_{m \in ts} \text{eCPM}_{t,p,m}$ then

Algorithm 2 [1] looks at each time period, and sorts the materials per placement by their eCPMs, highest to lowest. The function take With Budget Left takes an ordered set of materials M and an integer n as input; it returns the first available set $s \subseteq M$ where $|s| = n$ and each element of s is unique; and each eCPM of m order (m) for all $m \in s$ have sufficient budget left to be charged the current eCPM of m .

The algorithm parses the stream of impressions I_t , for each hour it picks the best creative material c , and a set of the highest eCPM text materials ts . The algorithm then picks either material c or the set of materials ts as the “winner” for the impression i by which has the highest eCPM total. The winner(s) are charged the eCPM to their budget, and the procedure is repeated until all impressions are parsed.

C. Comparison to production

The Baseline (Algorithm 2) is not an exact match of what is used in a production level ad exchange. There are a significant amount of constraints that the production algorithm considers that the Baseline does not. However, it does capture the most relevant aspects, and comparisons with the production system show that the revenue figures produced by the Baseline are in the same “ballpark” as the actual revenue that was realized.

V. OFF-LINE BUDGET OPTIMIZATION

This describes a linear programming (LP) model for doing off-line budget optimization [4][10]. The LP will provide a formal definition of the problem which this paper trying to solve. The solution of the LP will provide a revenue upper bound. It is known that the output of an LP is optimal in the context of the model, hence no hope of achieving higher revenue than the LP model. The use of linear programming for solving the off-line problem is not unheard of [3][4].

A. Linear programming

Linear programming [9] is a general method of formulating mathematical optimization problems. By defining an objective function that either want to maximize or minimize the value of; combined with a set of linear (in-)equality constraints, an LP solver can traverse the space of feasible solutions and locate an optimal solution, if one exists. The general definition of an LP[9] is

assign c to i order(c) $\leftarrow$ order(c) – eCPM$_{t,p,c}$ else for all $m \in ts$ do assign m to i order(m) $\leftarrow$ order(m) – eCPM$_{t,p,m}$ end for end if	maximize $\sum c \cdot x$ subject to $Ax \leq b$ and $x \geq 0$
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In this case x is the decision variable anybody wish to find an optimal assignment to; and c , b and A are parameters to the model. Note that the LP does not require x to be integer. In case, when x has the value of impressions nobody can possibly assign a fraction of an impression to a placement. However, the amount of impressions that it is working with makes the error from the relaxation insignificant.

B. Model input

The model this paper proposes develop aggregates impressions by hour. Also choose to work with “days”, i.e., 24 hour periods. Both these choices are easily manipulated, and the granularity of the impression aggregation and how long time-periods you work with is bounded only by the ever increasing combinatorial difficulty of the problem.

The parameters of the LP model [9] the model are

- The observed impressions per placement for a given publisher.
- Orders: orders are a set of materials, and a budget.
- Materials: a material can be either text or creative. A text material is a simple piece of plain text, and a creative is either an image or an Adobe Flash script.
- Placements: each placement can either only accept creative materials or text materials, or both. If a placement accepts text materials, it has a text material capacity defining how many text materials may fit on the same placement. This makes text materials special, since many text materials may share the same placement impression. Let placements that do not accept text materials have a text capacity of zero.
- An eCPM prediction model; rendering eCPM values for every hour $\times$ placement $\times$ material combination. It can use the eCPM values as a direct representation of publisher revenue.

C. Objective

This model has one decision variable x . x is a three dimensional matrix, and each element of x is a value for how many impressions are allocated to each hour $\times$ placement $\times$ material combination. Let T be the set of hours, M the set of materials, P the set of placements and v_t, p, m the eCPM for any combination of $t \in T, p \in P, m \in M$. The objective function for maximizing revenue is then formulated as maximize $\sum_{t,p,m} x_{t,p,m} v_{t,p,m}$

D. Constraints

- Let $I_{t,p}$ be the amount of impressions that placement $p \in P$ will receive at hour $t \in T$.
- Let an order $o \in O$ represents the materials belonging to that order.
- Let B_o be the budget for order o in the set of all orders O .

VI. ON-LINE BUDGET OPTIMIZATION

Recall the figure of the budget optimization system, also shown here. It has the necessary input available to define the on-line budget optimizing heuristic. Here define the “ROI- Finder” component in Figure 4 [1] as measuring a return-on- investment (ROI) value for orders totally consuming their budgets in some placement $\times$ material $\times$ time impression allocation S , and how it can use a matrix of such ROI values to adjust the eCPM of orders, to render higher revenue than the Baseline algorithm.

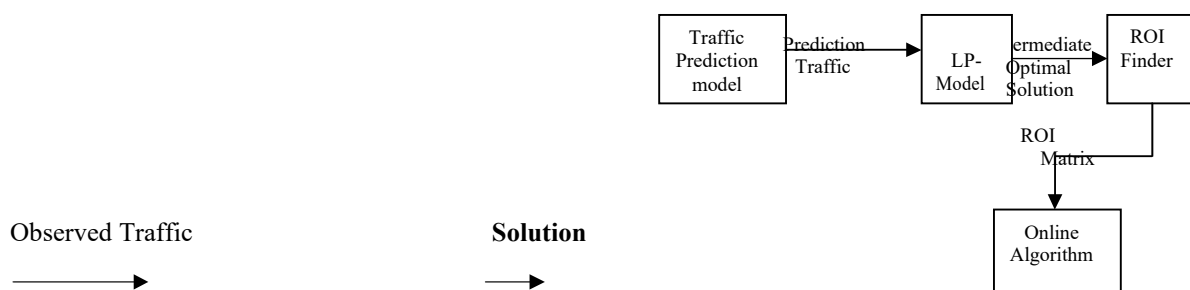


Fig 4: The budget optimization model.

To find an on-line heuristic [1] [8] that renders optimal or closer to optimal results compared to the Baseline algorithm presented above. Even with an optimal allocation S in hand, the information available to us is limited. S simply provides us with a static schedule of when, where and how many times to show a material. Such a



static schedule is not flexible enough to be useful in practice. Furthermore, in the production system, orders can enter and leave the system and bids can be changed. In this context, paper heuristic must be flexible enough to accommodate unforeseen changes still render higher revenue than the Baseline. The static allocation provided by S gives us some insight in how to do material selection, by looking at the difference in eCPM between the material that won impressions and the materials that did not.

A. Order classification

In the case that orders have unlimited, or infinite, budgets, it is always the case that for each impression one want to pick the highest eCPM materials since it do not have any budget constraints. On the other hand, if all orders were to consume their budgets, there is no further revenue to realize, as the resources are depleted. Hence, the interesting case is when the system is in such a state that some of the orders will consume their budgets and some will not. This allows the budget optimization model to reduce the number of impressions the orders that consume their budgets use, to free up impressions for the orders that have budget left. Papers classify orders as either budgeted or non-budgeted depending on if the order consumes its budget in S.

B. Return on investment (ROI)

As stated in the previous section, it have budgeted and non- budgeted orders. In this context, if a non-budgeted order P has the highest eCPM, it is always the optimal choice to let P win the impression. In the case that a budgeted order O has the highest eCPM, already it know that it may potentially be the case that O is better saved until a later impression, so as to consume O's budget in a more optimal fashion. In this sense, it can consider non-budgeted orders free of charge, as they are always optimal to show if they have the highest eCPM. Budgeted orders on the other hand should "motivate" that they are some measure "better" to choose now, rather than save their budget for a later, alternative impression. Call this measure return on investment (ROI). Only budgeted orders have ROI values, and it is possible to measure ROI values from any placement×material×time allocation S. For some budgeted order O, placement p and time t where O has won an impression, let P be the non-budgeted order with the highest eCPM[6] (among non-budgeted orders) $eCPM_{t,p}(\square) \square eCPM_{t,p}(\square)$ allocation S. These classify orders as budgeted and non- budgeted depending on if an order consumes its budget in S or not. From S show that it is possible to measure differences in eCPM between budgeted orders winning impressions and non-budgeted orders not winning impressions. These measurements can be used as a minimum return on investment requirement for budgeted orders in the on-line scenario, making these orders consume their budgets using fewer impressions than in the Baseline. The return on investment heuristic renders higher revenue compared to the Baseline, and helps the orders spend their budgets more evenly throughout the day and on the placements that produce the highest return on investment. The reason the revenue increases is because the budgeted orders consume their budgets using less impressions, allowing more impressions for non-budgeted orders, rendering an over-all increase in revenue for the publisher. If for some order o, time t and placement p, $ROI_{t,p}(o)$ is undefined, meaning that o has not won any impression for that time × placement combination, let pick the maximum value of the observed ROIs for order o as $ROI_{t,p}(o)$. This is motivated by the fact that o did not win impressions for time t and placement p in some allocation S, hence by using the maximum of the observed ROIs for o the heuristic will not "prefer" this t, p combination over any other combination. This is what makes the ROI heuristic flexible and able to handle changes in the ad exchange, as it allows the on-line algorithm to pick an unforeseen time × order × placement combination if it is good enough for us to disregard the allocation in S.

VII. CONCLUSION

This paper proposed a budget optimization model that has the potential to increase the revenue of publishers using ad exchanges by several percent. To achieve this, it further proposed a complete budget optimization model. It gives an LP model that solves the problem given off-line data. The LP model proves that there is potential for optimization, and also gives a revenue upper bound for any method in the same context as the model. Baseline algorithm that closely resembles the production algorithm, adapted to the simplified domain which consider in this paper. To make the LP useful for solving the on-line part of the problem paper proposed a linear regression model for predicting placement impressions. By combining the LP with the traffic prediction it can get a predicted optimal placement × material × time

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